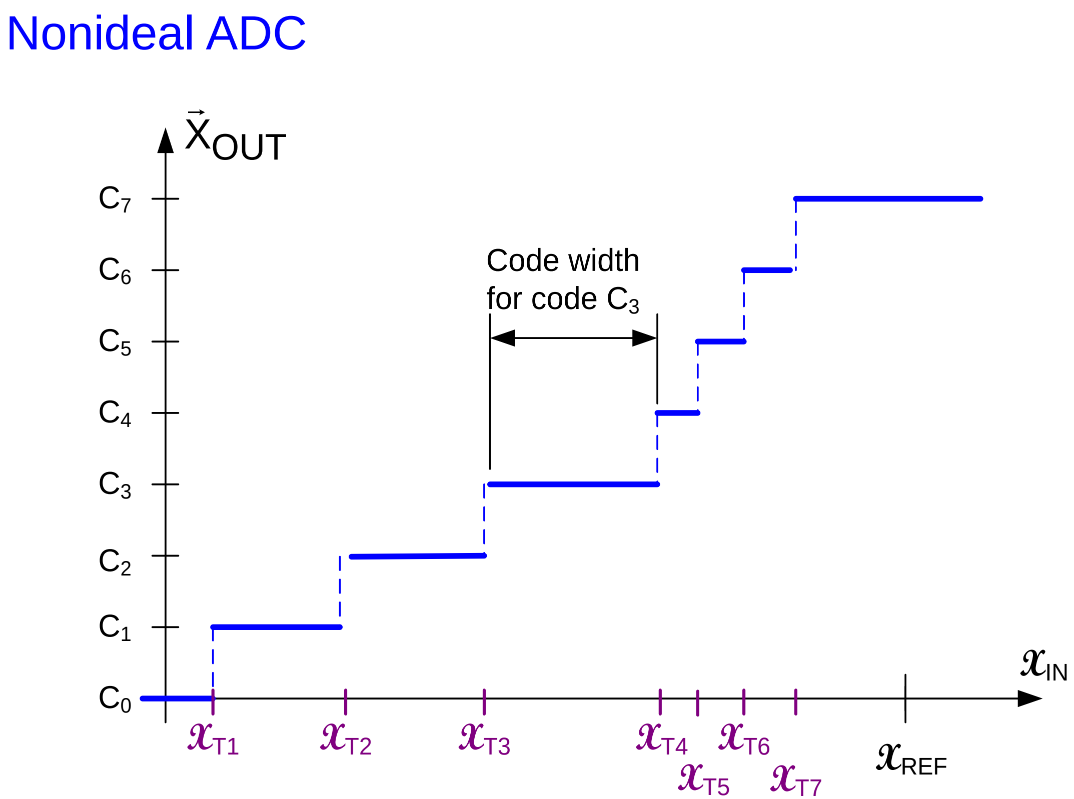
EE 505

Lecture 4

Quantization Noise
Spectral Characterization

Differential Nonlinearity (ADC)

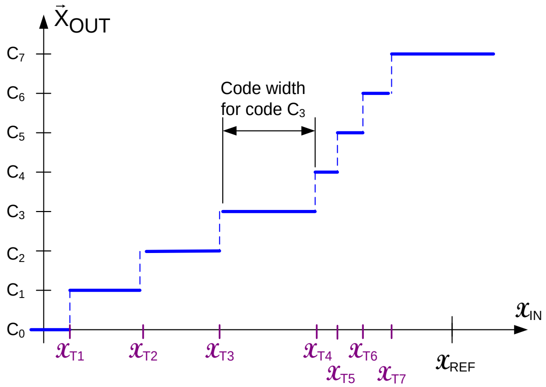


DNL(k) is the code width for code k – ideal code width normalized to X_{LSB}

$$DNL(k) = \frac{x_{T(k+1)} - x_{Tk} - x_{LSB}}{x_{LSB}}$$

Differential Nonlinearity (ADC)

Nonideal ADC



$$DNL(k)=\frac{x_{T(k+1)}-x_{Tk}-x_{LSB}}{x_{LSB}}$$

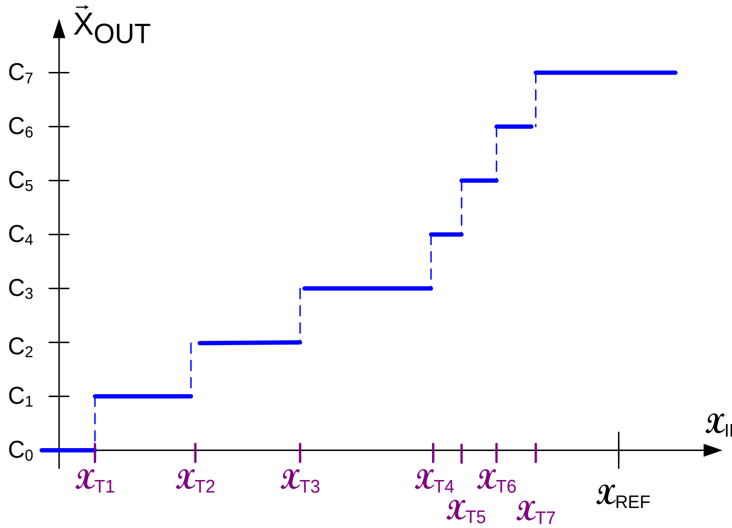
$$DNL=\max_{2\le k\le N-1}\left\{\left|DNL(k)\right|\right\}$$

DNL=0 for an ideal ADC

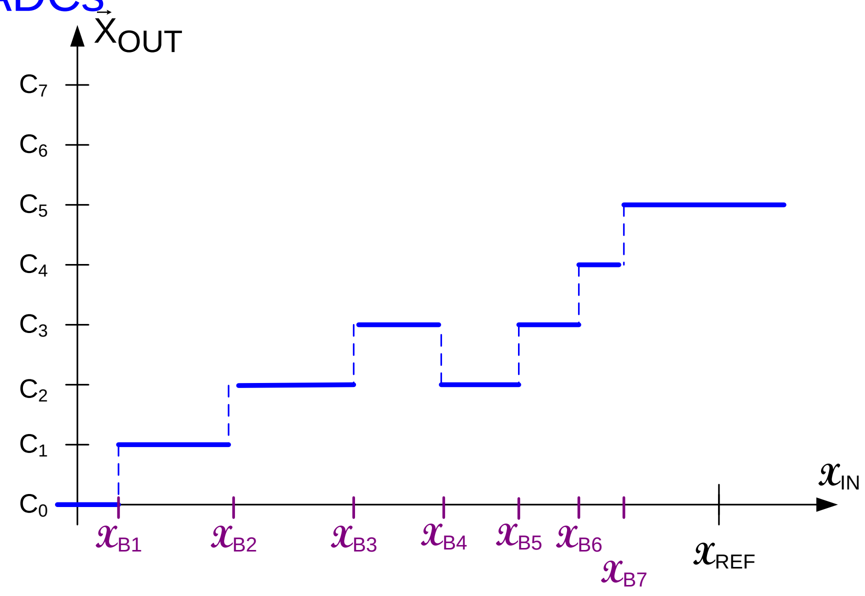
Note: In some nonideal ADCs, two or more break points could cause transitions to the same code C_k making the definition of DNL ambiguous

Monotonicity in an ADC

Nonideal ADCs



Monotone ADC



Nonmonotone ADC

Definition: An ADC is monotone if the

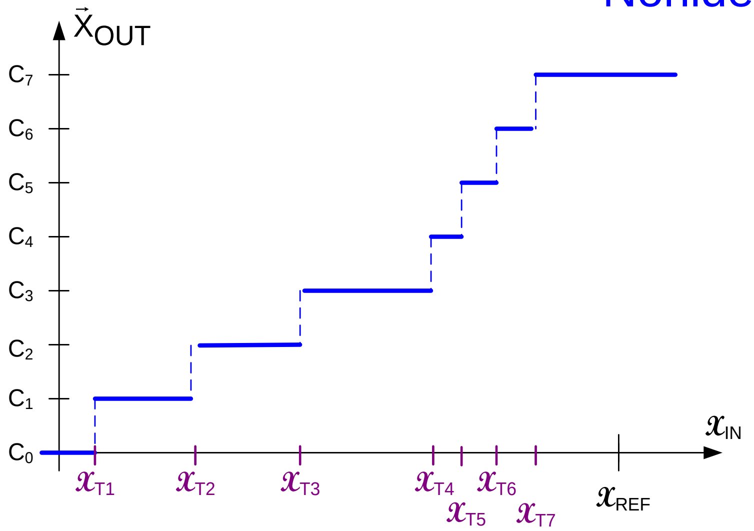
$$\vec{X}_{OUT}(x_k) \geq \vec{X}_{OUT}(x_m) \quad \text{whenever} \quad x_k \geq x_m$$

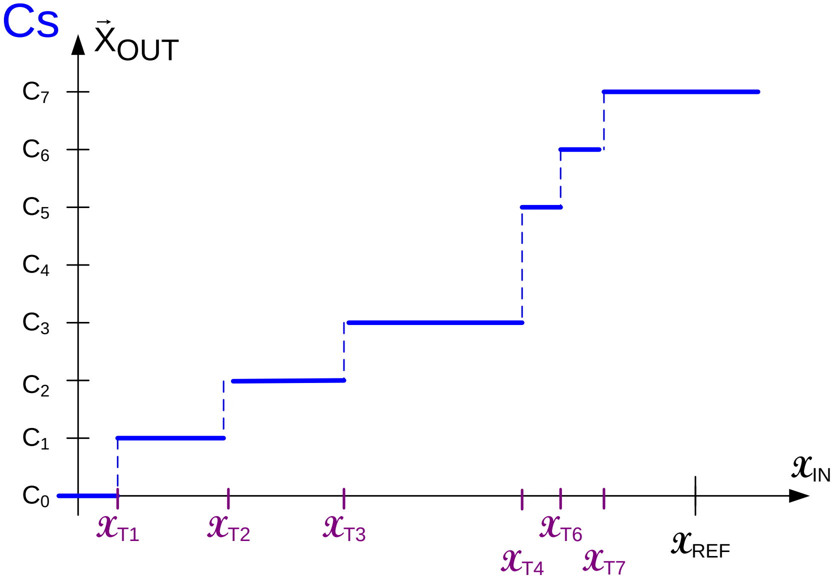
Note: Have used x_{Bk} instead of x_{Tk} in figure on right since more than one transition point corresponds to a given code

Note: Some authors do not define monotonicity in an ADC.

Missing Codes (ADC)

Nonideal ADCs





No missing codes

One missing code

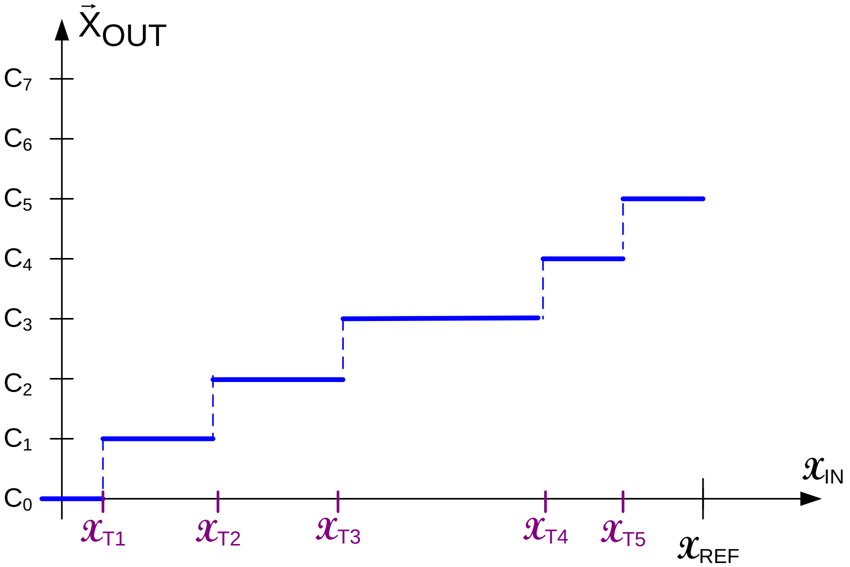
Definition: An ADC has no missing codes if there are N-1 transition points and a single LSB code increment occurs at each transition point. If these criteria are not satisfied, we say the ADC has missing code(s).

Note: With this definition, all codes can be present but we still say it has “missing codes”

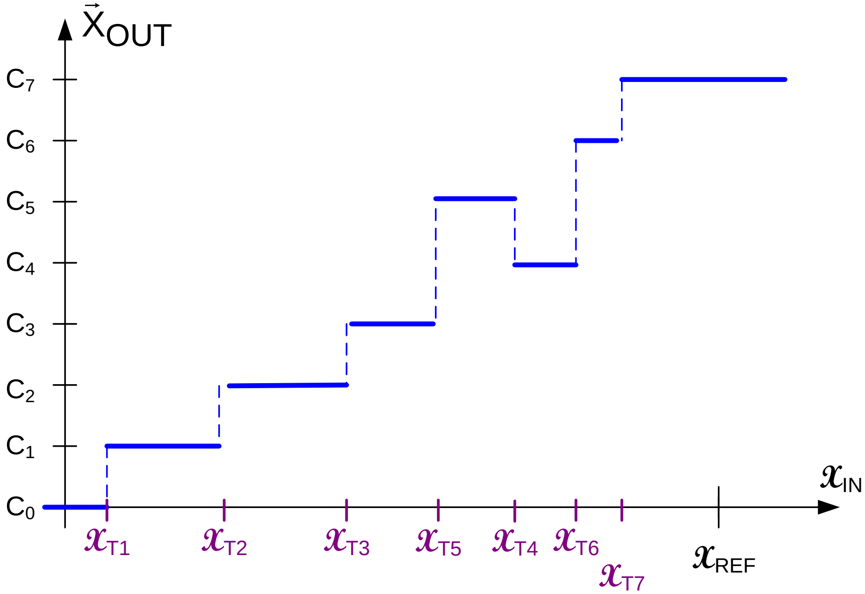
Note: Some authors claim that missing codes in an ADC are the counterpart to nonmonotonicity in a DAC. This association is questionable.

Missing Codes (ADC)

Nonideal ADCs



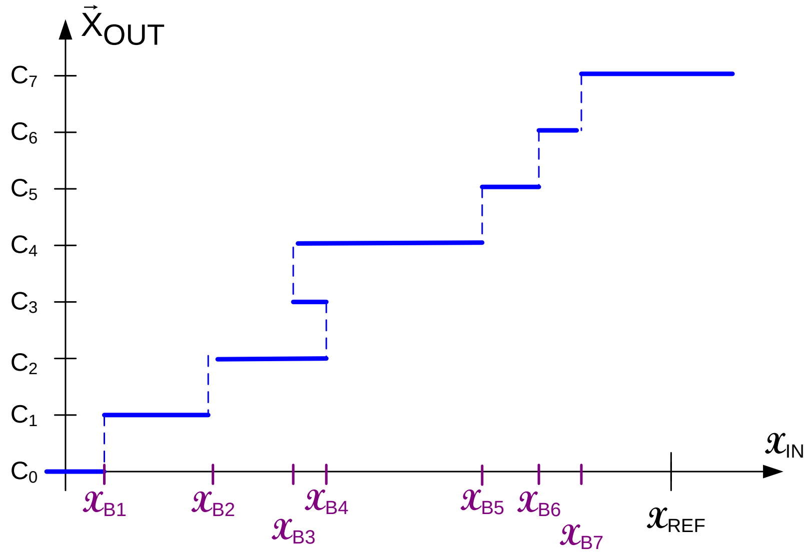
Missing codes

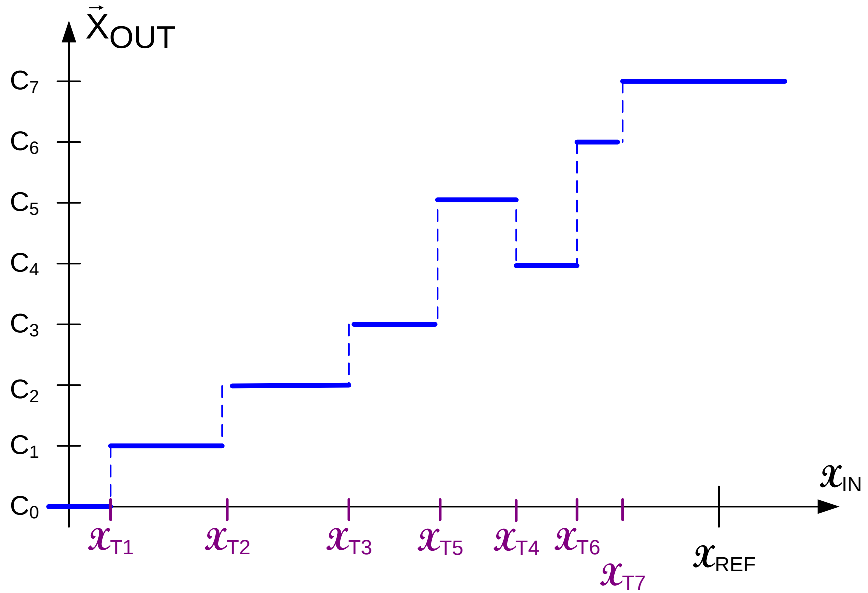


Missing code with all codes present

Weird Things Can Happen

Nonideal ADCs





- Multiple outputs for given inputs
- All codes present but missing codes

Be careful on definition and measurement of linearity parameters to avoid having weird behavior convolute analysis, simulation or measurements

Most authors (including manufacturers) are sloppy with their definitions of data converter performance parameters and are not robust to some weird operation

LSB Definition

X_{LSB} appears in many performance specifications but the definition of X_{LSB} is generally not given. This can cause modest inconsistencies in the definition of some performance specifications.

What is X_{LSB}?

LSB Definition

X_{LSB} appears in many performance specifications but the definition of X_{LSB} is generally not given. This can cause modest inconsistencies in the definition of some performance specifications.

What is X_{LSB}?

Conventional Wisdom X_{LSB}

$$X_{LSB} = \frac{X_{REF}}{2^{n_R}}$$

(X_{LSB} determined by specified resolution and can not be measured)

Alternate LSB Definition

X_{LSB} appears in many performance specifications but a distinction in X_{LSB} that differs from that obtained from specified values for X_{REF} and n_R is generally not given. This can cause modest inconsistencies in the definition of some performance specifications.

DAC

Alternate definitions of X_{LSB}

where N is the measured number of DAC output levels

$$X_{LSB}=\frac{X_{REF}}{N}$$

---

where N is the measured number of DAC output levels and $X_0(N-1)$ and $X_0(0)$ are last and first outputs respectively

$$X_{LSB}=\frac{X_0(N-1)-X_0(0)}{N-1}$$

---

useful when extreme values do not occur at minimum and maximum input codes

$$X_{LSB}=\frac{\max\limits_k\{X_0(k)\}-\min\limits_k\{X_0(k)\}}{N-1}$$

---

useful for determining worst-case resolution of a DAC

$$X_{LSB}=\max_k\{X_0(k)-X_0(k-1)\}$$

ADC

Similar definitions can be made for X_{LSB} of an ADC based upon the breakpoints

Alternate LSB Definition

Is the concept of an LSB that is based upon measurements useful?

In many control applications, the largest gap between outputs of a DAC is often of interest and though that is ideally V_{LSB}, it may differ significantly

ENOB based upon DNL

If it is assumed that an acceptable DNL for an n-bit data converter is $X_{LSB}/2$, then if the DNL is different from $X_{LSB}/2$, the effective number of bits essentially changes.

An ENOB based upon the DNL can be defined (homework problem)

ENOB relative to resolution

Summary of previous observations relating to ENOB (based upon INL) :

If an n-bit data converter has an INL of ¼ LSB, it is really performing from a linearity viewpoint at the n+1 bit level and if it has an INL of 1/8 LSB it is really performing at the n+2 bit level

Correspondingly, if it has a DNL of ¼ LSB, it is also performing from a differential linearity viewpoint at the n+1 bit level

The ENOB (based upon INL) of a data converter can exceed the number of bits of resolution of the data converter

Some applications benefit from an ENOB that exceeds the resolution of the data converter

Limitations of INL & DNL in Characterizing Linearity

Will discuse later

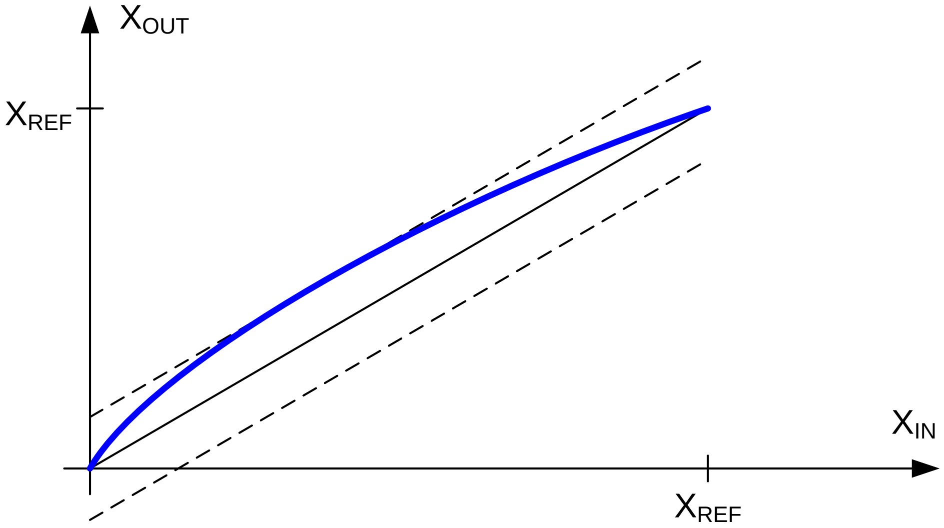
- **INL is a key parameter that is attempting to characterize the overall linearity of a DAC !**
- **INL is a key parameter that is attempting to characterize the overall linearity of an ADC !**
- **DNL is a key parameter that is attempts to characterize the local linearity of a DAC !**
- **DNL is a key parameter that is attempts to characterize the local linearity of an ADC !**

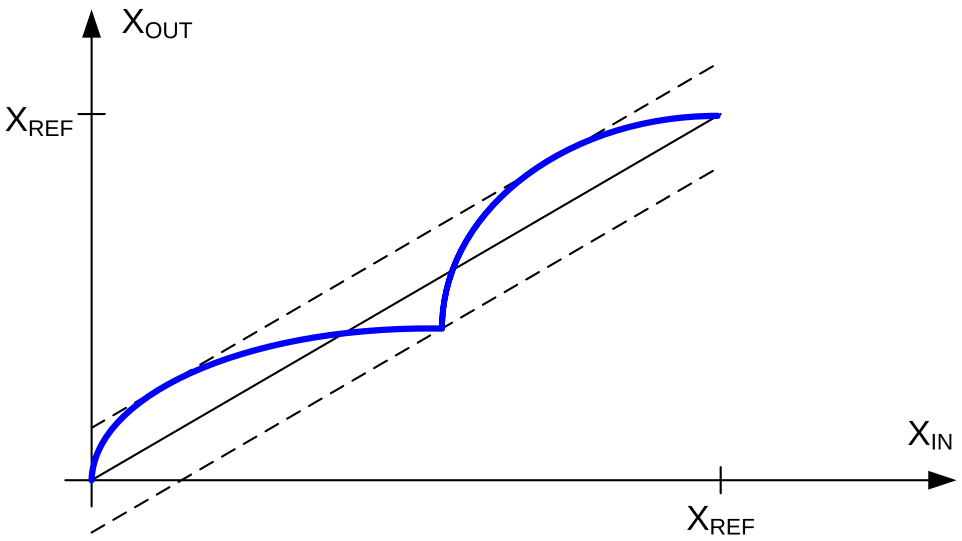
Are INL and DNL effective at characterizing the linearity of a data converter?

Limitations of INL & DNL in Characterizing Linearity

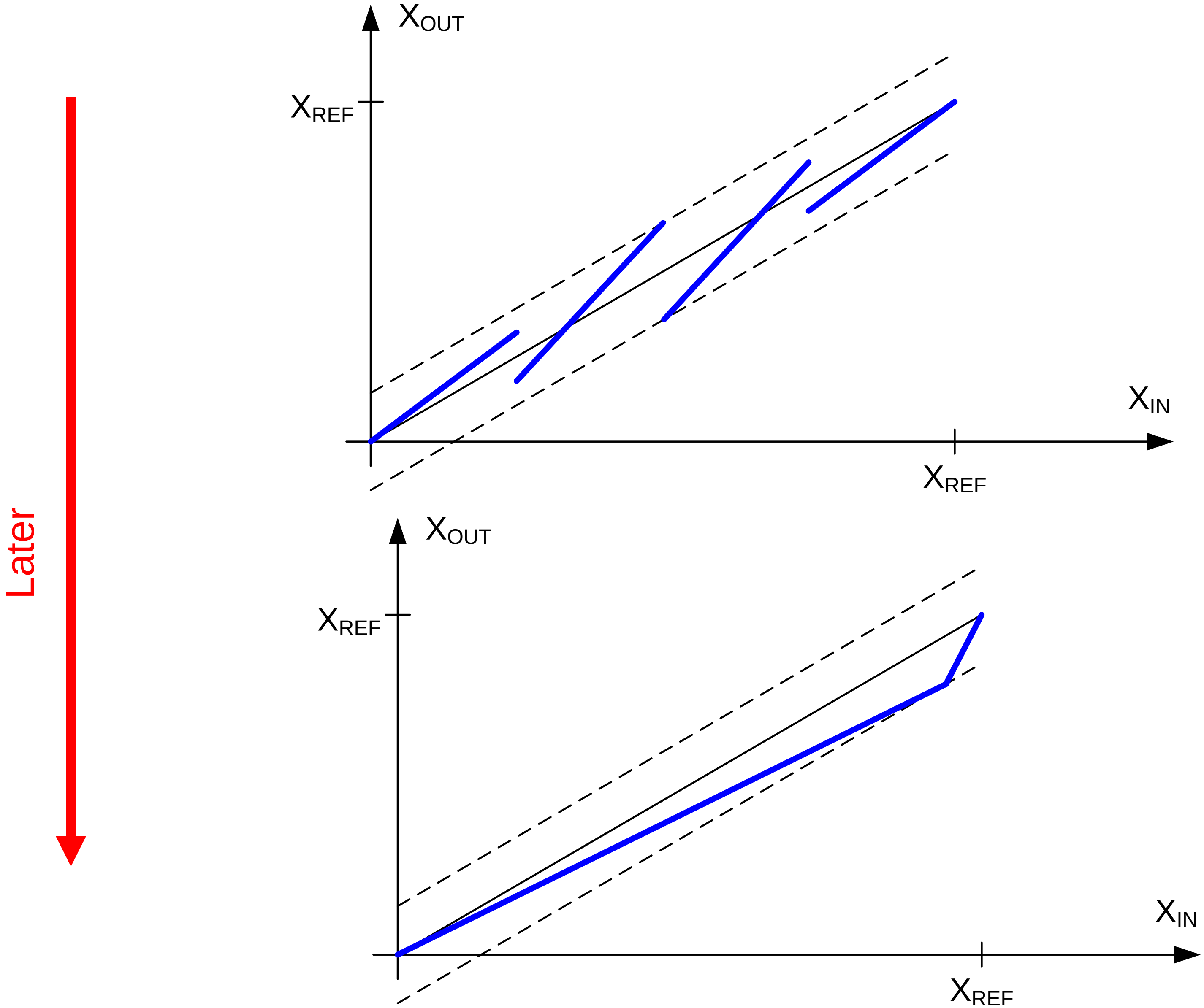
Consider the following 4 transfer characteristics, all of which have the same INL

Later

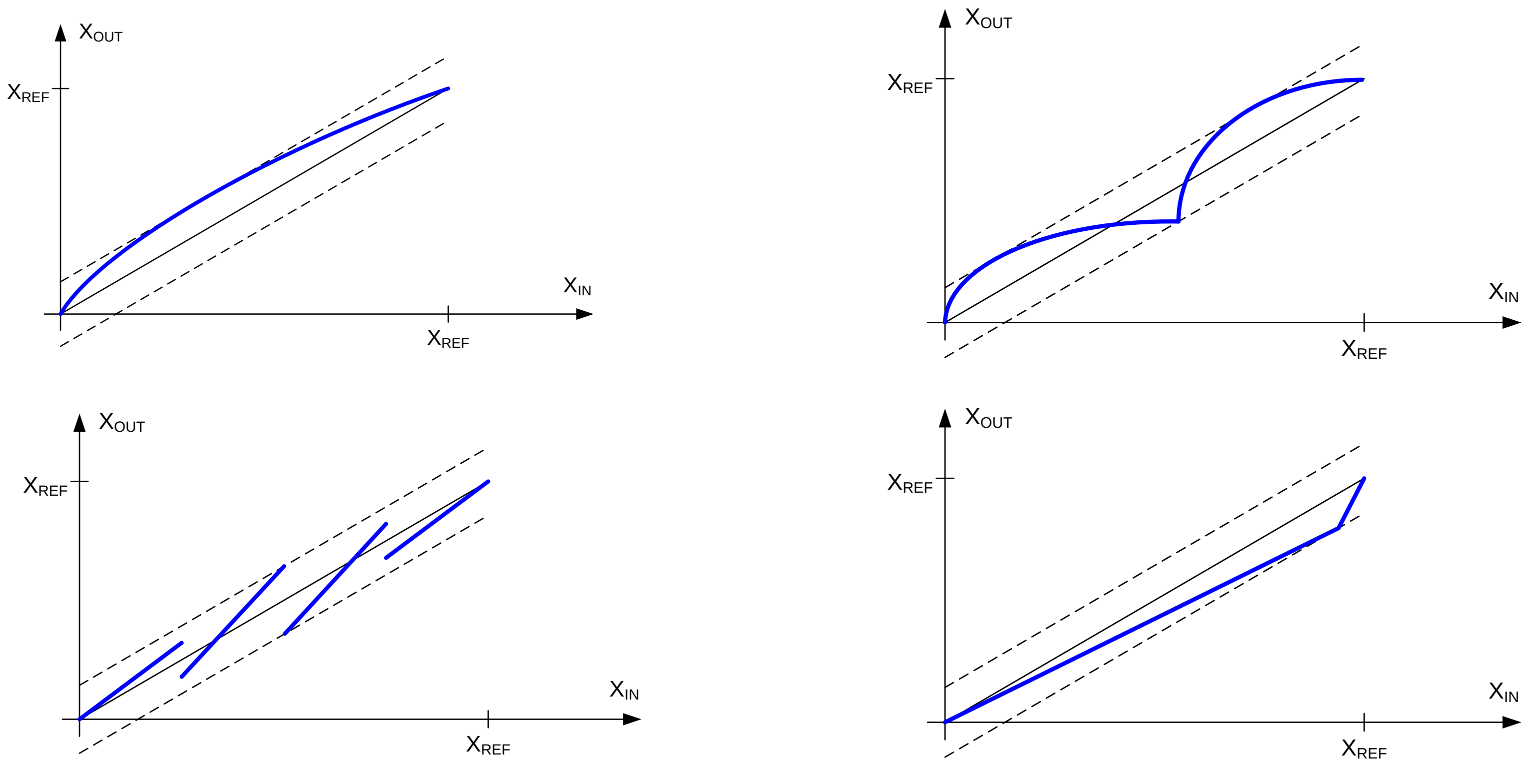




Limitations of INL & DNL in Characterizing Linearity



Limitations of INL & DNL in Characterizing Linearity



Later

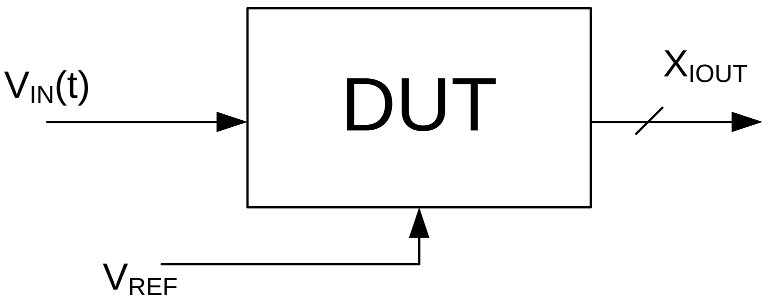
Although same INL, dramatic difference in performance particularly when inputs are sinusoidal-type excitations

INL also gives little indication of how performance degrades at higher frequencies

Spectral Analysis often used as an alternative (and often more useful in many applications) linearity measure for data converters

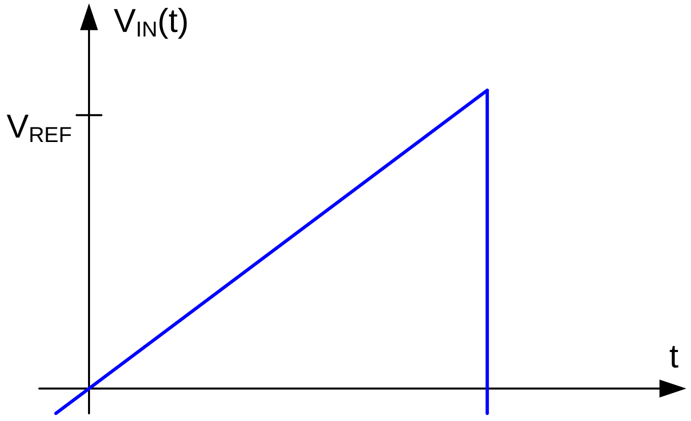
Linearity Measurements (testing)

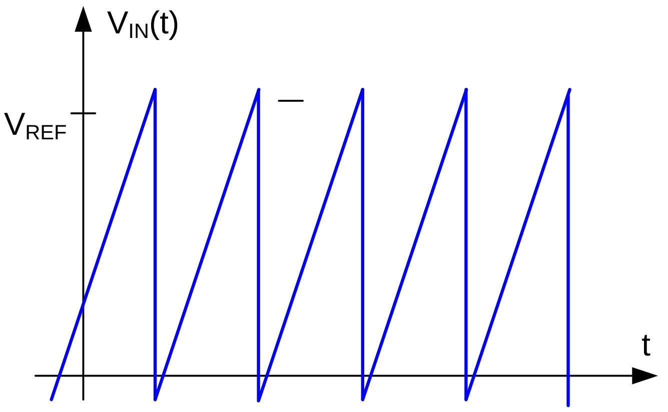
Consider ADC



Linearity testing often based upon code density testing

Code density testing:



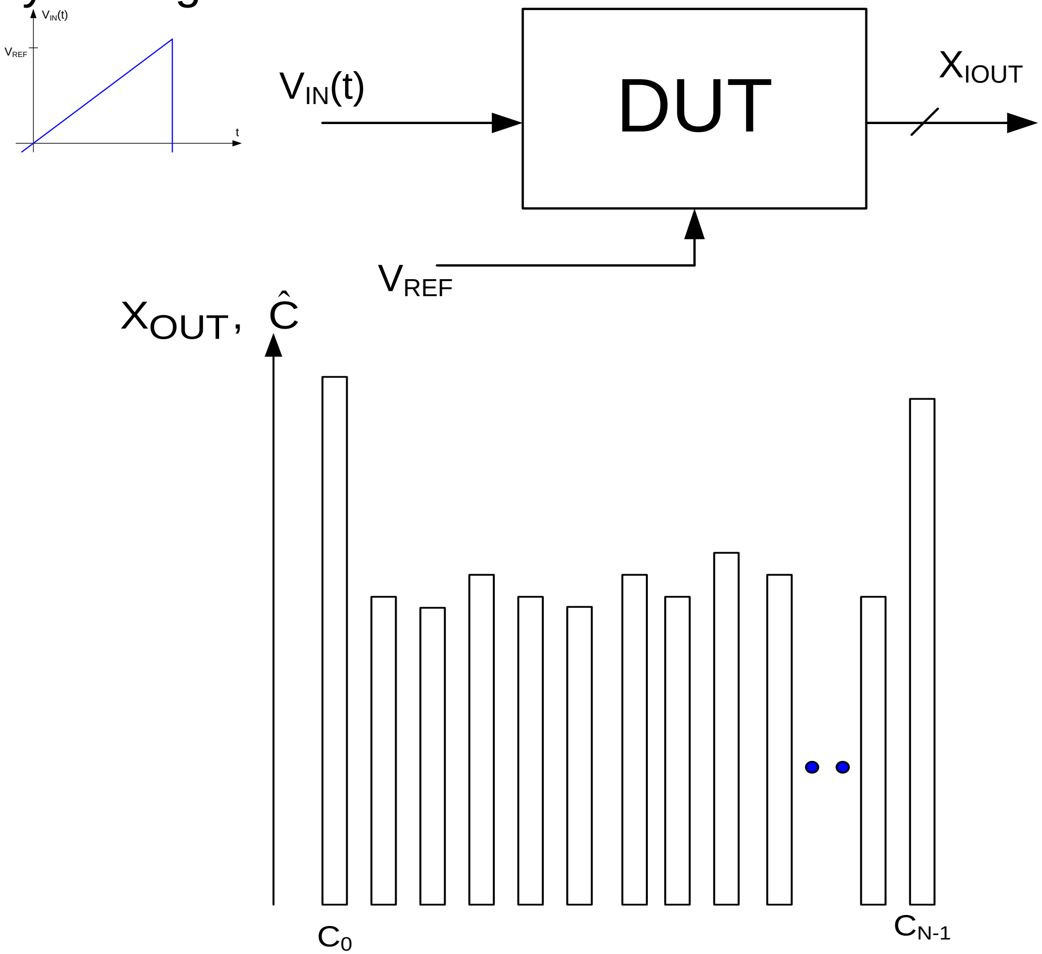


Ramp or multiple ramps often used for excitation

Linearity of test signal is critical (typically 3 or 4 bits more linear than DUT)

Linearity Measurements (testing)

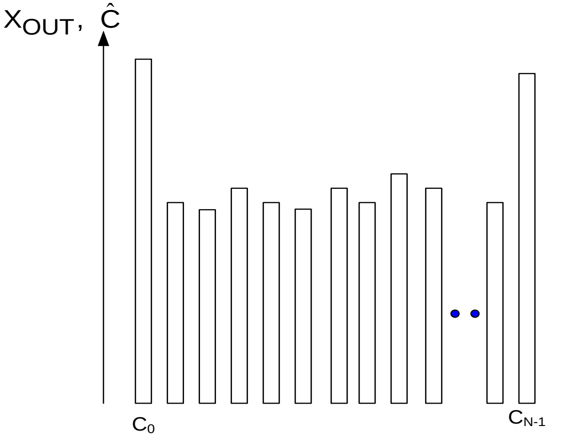
Code density testing:



- First and last bins generally have many extra counts (and thus no useful information)
- Typically average 16 or 32 hits per code

Linearity Measurements (testing)

Code density testing:



$$\bar{C} = \frac{\sum_{i=1}^{N-2} \hat{C}_i}{N-2}$$

$$DNL_i = \frac{\hat{C}_i - \bar{C}}{\bar{C}}$$

$$INL_i = \begin{cases} 0 & i=0,N-2 \\ \dfrac{\left[\sum_{k=1}^{i} \hat{C}_k\right] - i\bar{C}}{\bar{C}} & 1 \le i \le N-3 \end{cases}$$

$$DNL = \max_{1 \le i \le N-2} \left\{ \left| DNL_i \right| \right\}$$

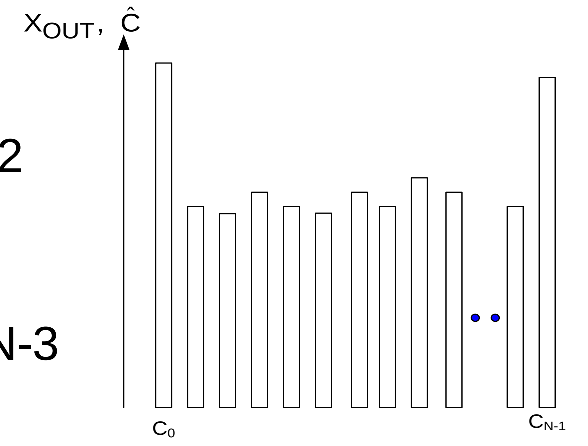
$$INL = \max_{1 \le i \le N-3} \left\{ \left| INL_i \right| \right\}$$

Linearity Measurements (testing)

Code density testing:

$$DNL_i = \frac{\hat{C}_i - \bar{C}}{\bar{C}} \qquad INL_i = \begin{cases} 0 & i=0,N-2 \\ \dfrac{\left[\sum_{k=1}^{i} \hat{C}_k\right] - i\bar{C}}{\bar{C}} & 1 \le i \le N-3 \end{cases}$$

$$DNL = \max_{1\le i\le N-2}\left\{\left|DNL_i\right|\right\} \qquad INL = \max_{1\le i\le N-3}\left\{\left|INL_i\right|\right\}$$



- **Code Density Measurements are Indirect Measurements of the INL and DNL**
- **Can give very wrong information under some nonmonotone missing code scenarios**
- **Often use an average of 16 or 32 samples per code**
- **Measurement noise often 1 lsb or larger but averages out**
- **Sometimes use good sinusoidal waveform but must correct code density for this distinction**
- **Full code-density testing is costly for high-resolution low-speed data converters because of data acquisition costs**
- **Reduced code testing using servo methods is often a less costly alternative but may miss some errors**

Performance Characterization of Data Converters

- Static characteristics
  - Resolution
  - Least Significant Bit (LSB)
  - Offset and Gain Errors
  - Absolute Accuracy
  - Relative Accuracy
  - Integral Nonlinearity (INL)
  - Differential Nonlinearity (DNL)
  - Monotonicity (DAC)
  - Missing Codes (ADC)
  - Quantization Noise
  - Low-f Spurious Free Dynamic Range (SFDR)
  - Low-f Total Harmonic Distortion (THD)
  - Effective Number of Bits (ENOB)
  - Power Dissipation

Quantization Noise

- DACs and ADCs generally quantize both amplitude and time
- If converting a continuous-time signal (ADC) or generating a desired continuous-time signal (DAC) these quantizations cause a difference in time and amplitude from the desired signal
- First a few comments about Noise

What is Noise in a data converter?

Noise is a term applied to some nonideal effects of a data converter

Precise definition of noise is probably not useful

Some differences in views about what nonideal characteristics of a data converter should be referred to as noise

Types of noise:

- Random perturbations in V or I due to movement of electrons in electronic devices
- Interfering signals generated by other systems
- Interfering signals generated by a circuit or system itself
- Error signals associated with imperfect signal processing algorithms or circuits
  - Quantization noise
  - Sample Jitter
  - Harmonic Distortion

Noise

All of these types of noise are present in data converters and are of concern when designing most data converters

Can not eliminate any of these noise types but with careful design can manage their effects to certain levels

Noise (in particular the random noise) is often the major factor limiting the ultimate performance potential of many if not most data converters

Noise

Types of noise:

- Perturbations in V or I due to movement of electrons in electronic circuits
- Interfering signals generated by other systems
- Interfering signals generated by a circuit or system itself
- Error signals associated with imperfect signal processing algorithms or circuits
  - Quantization noise
  - Sample Jitter
  - Harmonic Distortion

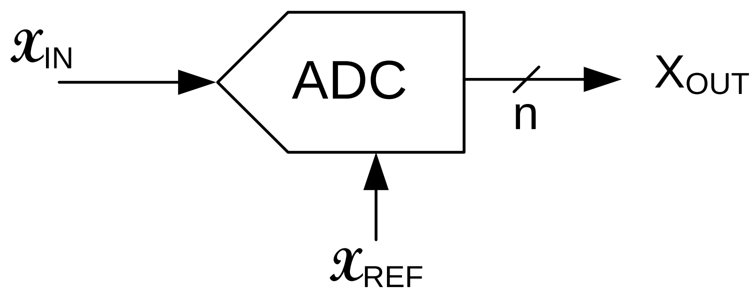
Quantization noise is a significant component of this type of “noise” in ADCs and DACs and is present even if the ADC or DAC is ideal

Only the first type is associated with random variations but from a performance limitation viewpoint, all appear to adversely affect the desired output in similar ways

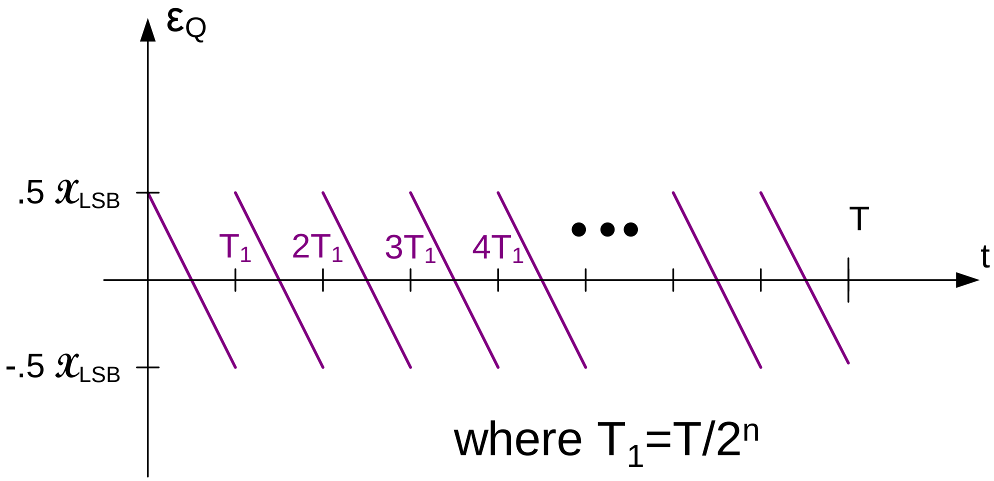
Quantization Noise in ADC

(same concepts apply to DACs)

Consider an Ideal ADC with first transition point at $0.5X_{LSB}$



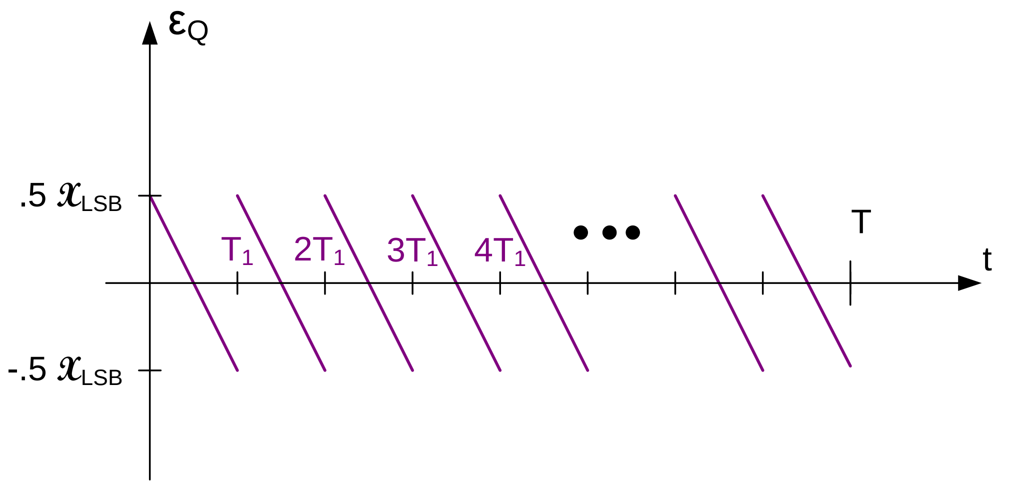
If the input is a low frequency sawtooth waveform of period T that goes from 0 to X_{REF} , the error signal in the time domain will be:



where $T_1 = T/2^n$

This time-domain waveform (after dc offset is removed) is termed the Quantization Noise for the ADC with a sawtooth (or triangular) input

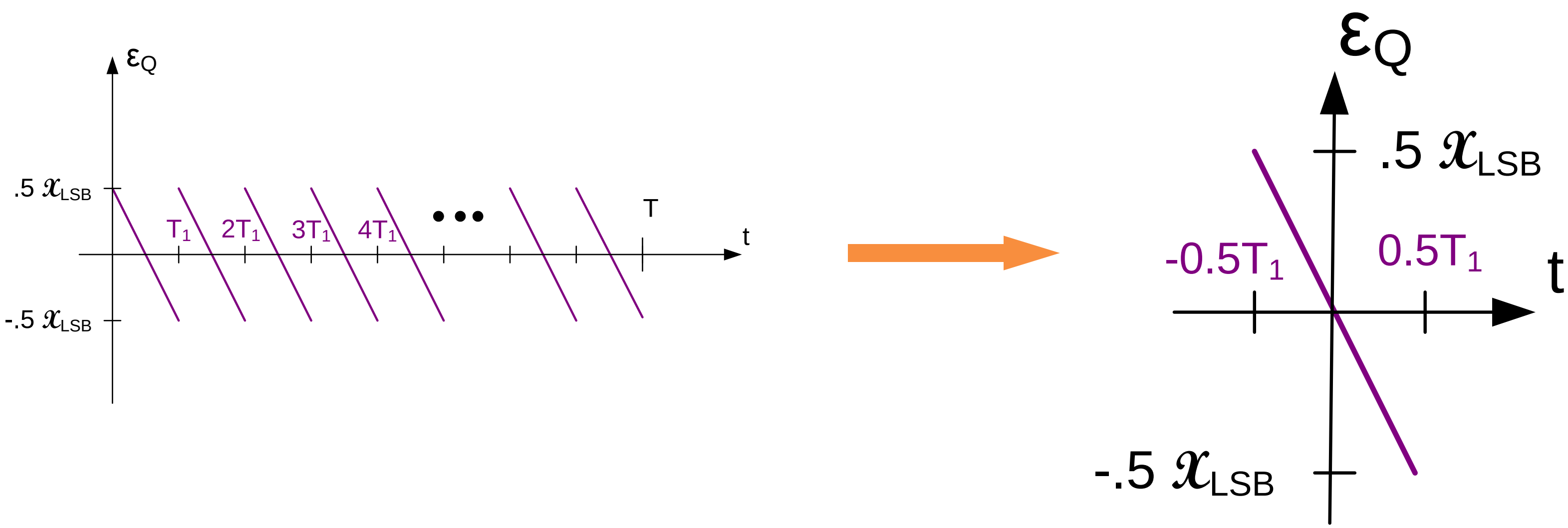
Quantization Noise in ADC



For large n this periodic waveform "behaves" much like a random noise source that is uncorrelated with the input and can be characterized by its RMS value which can be obtained by integrating over any interval of length T_1. For notational convenience, shift the waveform to the left by $T_1/2$ units

$$E_{RMS} = \sqrt{\frac{1}{T_1} \int_{-T_1/2}^{T_1/2} \varepsilon_Q{}^2(t)\,dt}$$

Quantization Noise in ADC



$$E_{RMS} = \sqrt{\frac{1}{T_1}\int_{-T_1/2}^{T_1/2} \varepsilon_Q{}^2(t)\,dt}$$

In this interval, ε_Q can be expressed as

$$\varepsilon_Q(t) = -\left(\frac{X_{LSB}}{T_1}\right)t$$

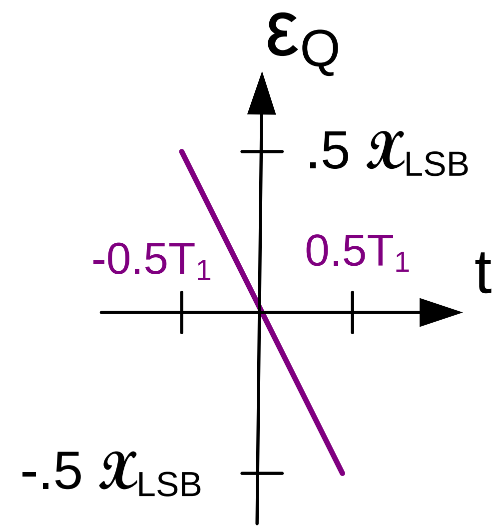
Quantization Noise in ADC

$$E_{RMS} = \sqrt{\frac{1}{T_1}\int_{-T_1/2}^{T_1/2} \varepsilon_Q{}^2(t)\,dt}$$

$$E_{RMS} = \sqrt{\frac{1}{T_1}\int_{-T_1/2}^{T_1/2} \left(-\frac{x_{LSB}}{T_1}\right)^2 t^2\,dt}$$

$$E_{RMS} = x_{LSB}\sqrt{\frac{1}{T_1^3}\left.\frac{t^3}{3}\right|_{-T_1/2}^{T_1/2}}$$

$$E_{RMS} = \frac{x_{LSB}}{\sqrt{12}}$$



$$\varepsilon_Q(t) = -\left(\frac{X_{LSB}}{T_1}\right)t$$

Quantization Noise in ADC

$$E_{RMS} = \frac{x_{LSB}}{\sqrt{12}}$$

The signal to quantization noise ratio (SNR) can now be determined. Since the input signal is a sawtooth waveform of period T and amplitude X_{REF}, it follows by the same analysis that it has an RMS value of

$$x_{RMS} = \frac{x_{REF}}{\sqrt{12}}$$

Thus the SNR is given by

$$SNR = \frac{x_{RMS}}{E_{RMS}} = \frac{x_{RMS}}{x_{LSB}} = 2^n$$

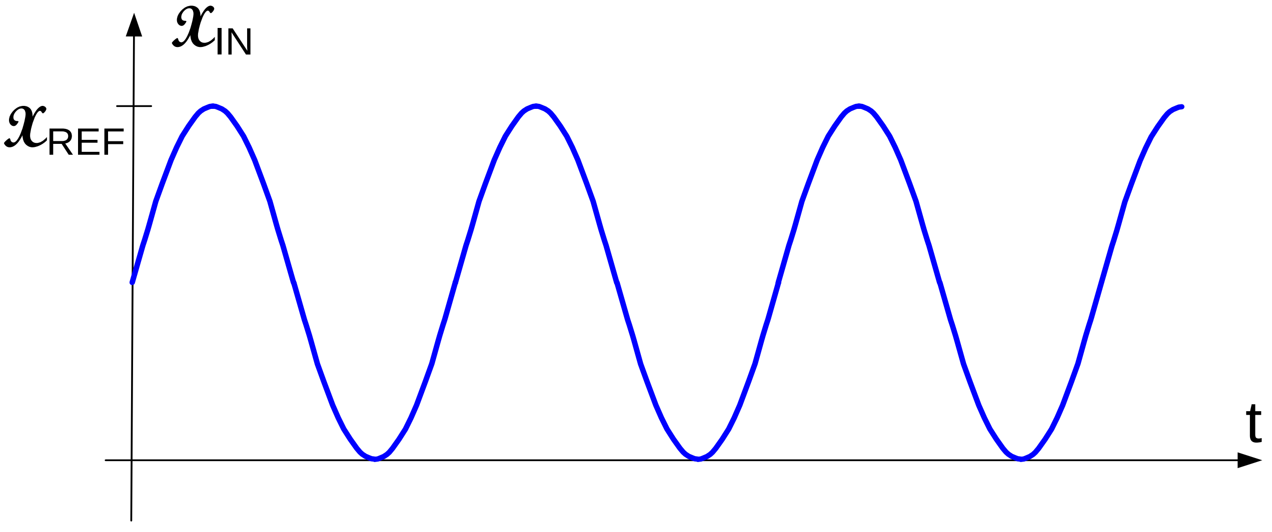
or, in dB,

$$SNR_{dB} = 20\left(n \bullet \log 2\right) = 6.02n$$

Note: dB subscript often neglected when not concerned about confusion

Quantization Noise in ADC

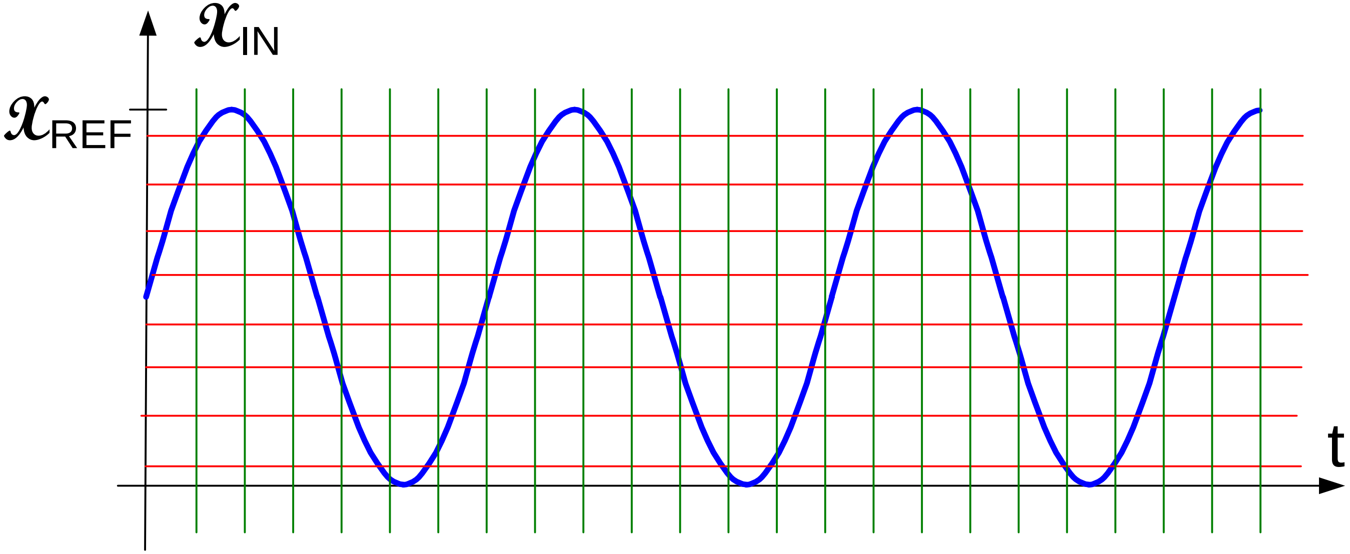
How does the SNR change if the input is a sinusoid that goes from 0 to x_{REF} centered at $x_{REF}/2$?



$$SNR = 20(n \bullet \log 2) =? \ 6.02n$$

Quantization Noise in ADC

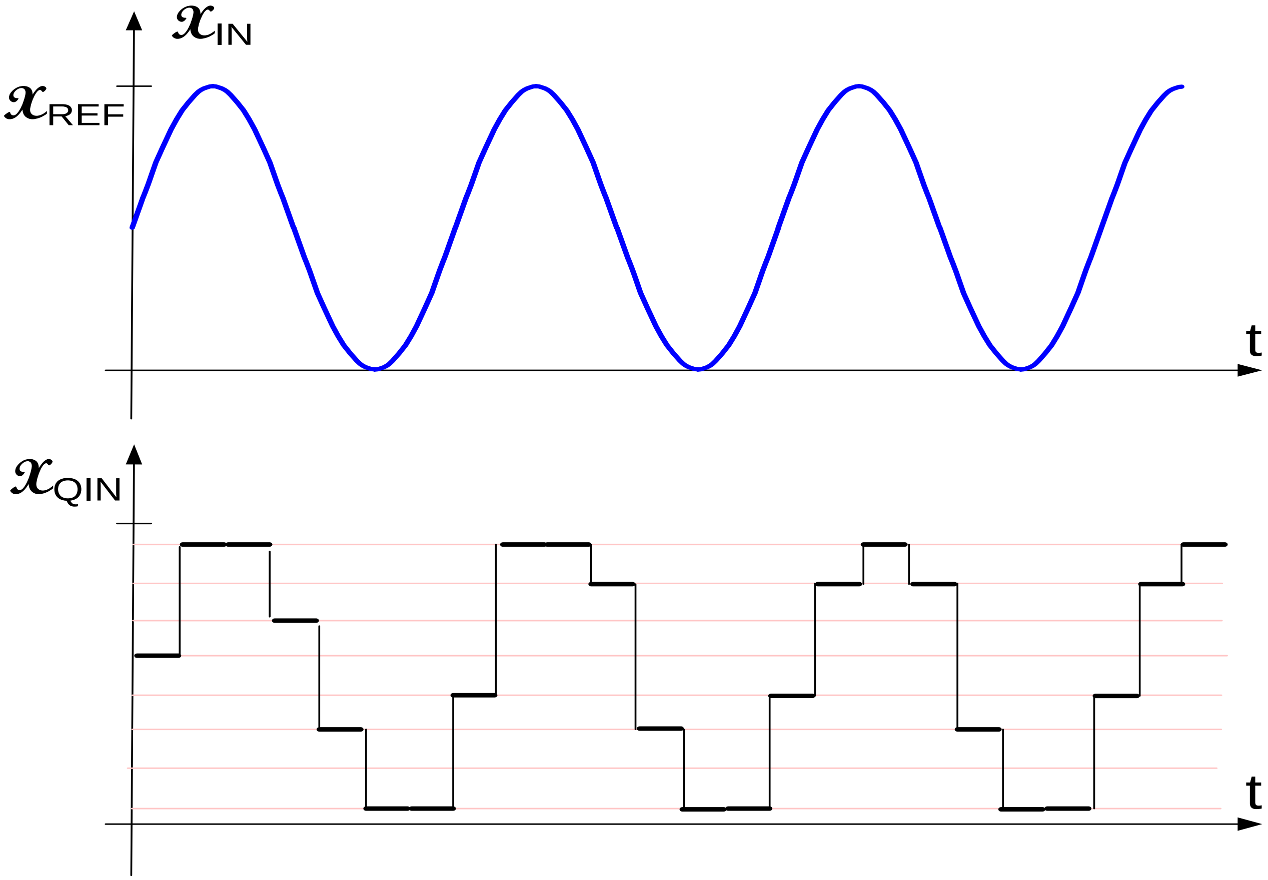
How does the SNR change if the input is a sinusoid that goes from 0 to x_{REF} centered at $x_{REF}/2$?



Time and amplitude quantization points

Quantization Noise in ADC

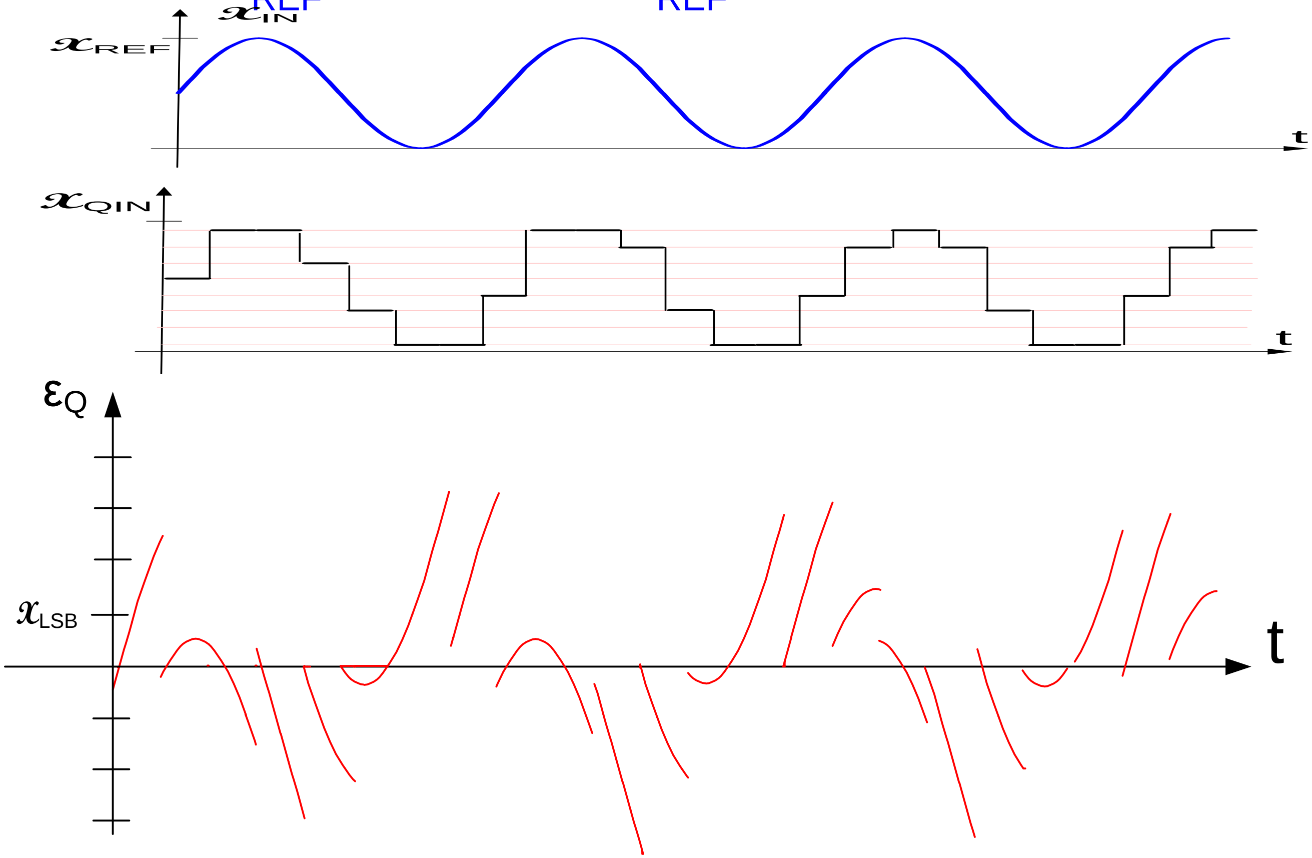
How does the SNR change if the input is a sinusoid that goes from 0 to x_{REF} centered at $x_{REF}/2$?



Time and Amplitude Quantized Waveform

Quantization Noise in ADC

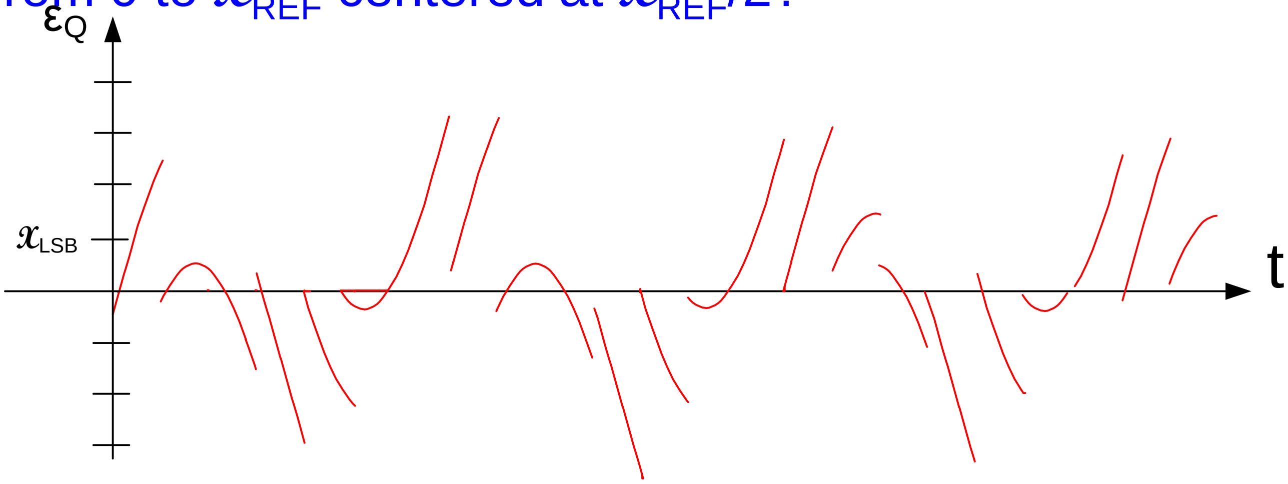
How does the SNR change if the input is a sinusoid that goes from 0 to x_{REF} centered at $x_{REF}/2$?



Error waveform

Quantization Noise in ADC

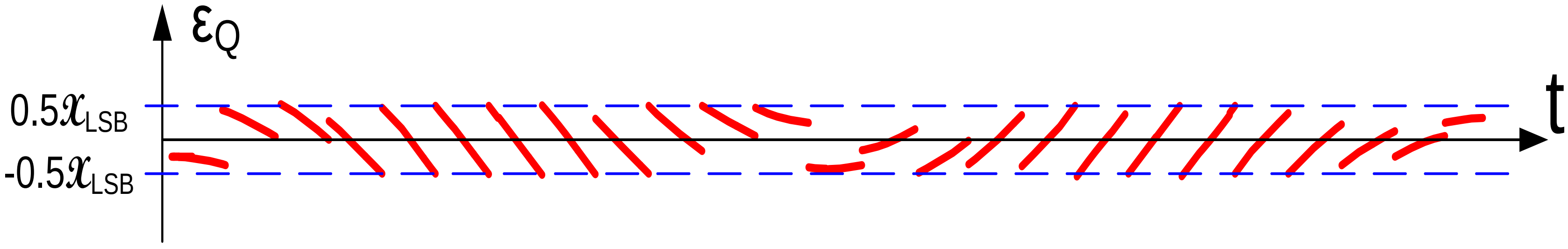
How does the SNR change if the input is a sinusoid that goes from 0 to $\mathcal{X}_{REF}$ centered at $\mathcal{X}_{REF}/2$?



- Appears to be highly uncorrelated with input even though deterministic
- Mathematical expression for ε_Q very messy
- Excursions exceed X_{LSB} (but will be smaller and bounded by ± $X_{LSB}/2$ for lower frequency signal/frequency clock ratios)
- For lower frequency inputs and higher resolution, at any time, errors are approximately uniformly distributed between $-X_{LSB}/2$ and $X_{LSB}/2$
- Analytical form for ε_{QRMS} essentially impossible to obtain from $\varepsilon_Q(t)$

Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $\mathcal{X}_{REF}$ centered at $\mathcal{X}_{REF}/2$?



For low f_{SIG}/f_{CL} ratios, bounded by ±XLB and at any point in time, behaves almost as if a uniformly distributed random variable

$$\varepsilon_Q \sim U[-0.5X_{LSB}, 0.5X_{LSB}]$$

Quantization Noise in ADC

Recall:

If the random variable f is uniformly distributed in the interval [A,B]
f : U[A,B]   then the mean and standard deviation of f are given by

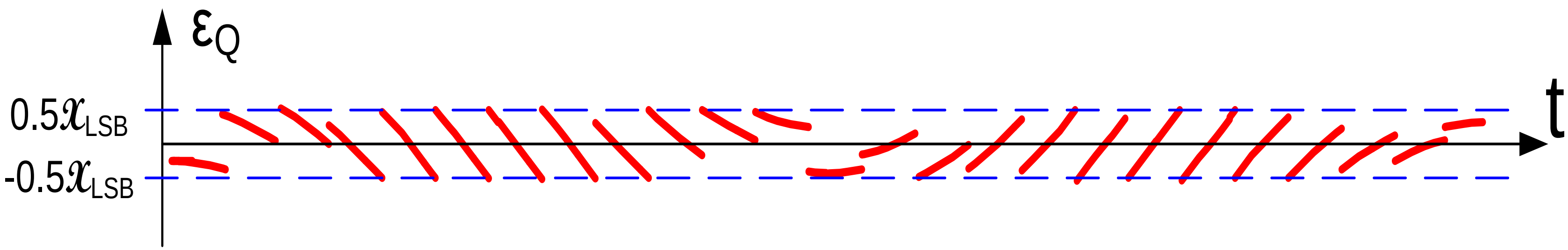
$$\mu_f = \frac{A+B}{2} \qquad \sigma_f = \frac{B-A}{\sqrt{12}}$$

Theorem: If n(t) is a random process, then for large T,

$$V_{RMS} = \sqrt{\frac{1}{T}\int_{t_1}^{t_1+T} n^2(t)\,dt} = \sqrt{\sigma_n^2 + \mu_n^2}$$

Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $\mathcal{X}_{REF}$ centered at $\mathcal{X}_{REF}/2$?



$$\varepsilon_Q \sim U[-0.5X_{LSB}, 0.5X_{LSB}]$$

$$\mu_{\varepsilon_Q} = \frac{A+B}{2} = 0 \qquad \sigma_f = \frac{B-A}{\sqrt{12}} = \frac{X_{LSB}}{\sqrt{12}}$$

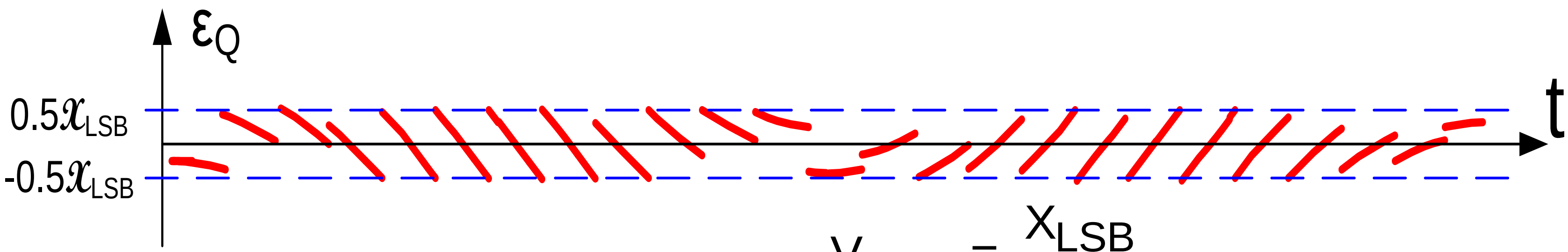
$$V_{RMS} = \sqrt{\frac{1}{T}\int_{t_1}^{t_1+T} n^2(t)\,dt} = \sqrt{\sigma_n^2 + \mu_n^2}$$

$$V_{RMS} = \sigma_{\varepsilon_Q} = \frac{X_{LSB}}{\sqrt{12}}$$

Note this is the same RMS noise that was present with a triangular input

Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $\mathcal{X}_{REF}$ centered at $\mathcal{X}_{REF}/2$?



$$V_{RMS} = \frac{X_{LSB}}{\sqrt{12}}$$

But $$V_{INRMS} = \left(\frac{X_{REF}}{2}\right)\frac{1}{\sqrt{2}}$$

Thus obtain

$$SNR = \frac{\frac{X_{REF}}{2\sqrt{2}}}{\frac{X_{LSB}}{\sqrt{12}}} = 2^n\sqrt{\frac{3}{2}}$$

Finally, in db,

$$SNR_{dB} = 20\log\left(2^n\sqrt{\frac{3}{2}}\right) = 6.02\,n + 1.76$$

ENOB based upon Quantization Noise

$$SNR = 6.02\,n + 1.76$$

Solving for n, obtain

$$ENOB = \frac{SNR_{dB} - 1.76}{6.02}$$

Note: could have used the SNR_{dB} for a triangle input and would have obtained the expression

$$ENOB = \frac{SNR_{dB}}{6.02}$$

But the earlier expression is more widely used when specifying the ENOB based upon the noise level present in a data converter

ENOB based upon Quantization Noise

For very low resolution levels, the assumption that the quantization noise is uncorrelated with the signal is not valid and the ENOB expression will cause a modest error

from van de Plassche (p13)

$$SNR_{corr} \cong \left(2^n - 2 + \frac{4}{\pi}\right)\sqrt{\frac{3}{2}}$$

Res (n)	SNR_{corr}	SNR
1	3.86	7.78
2	12.06	13.8
3	19.0	19.82
4	25.44	25.84
5	31.66	31.86
6	37.79	37.88
8	49.90	49.92
10	61.95	61.96

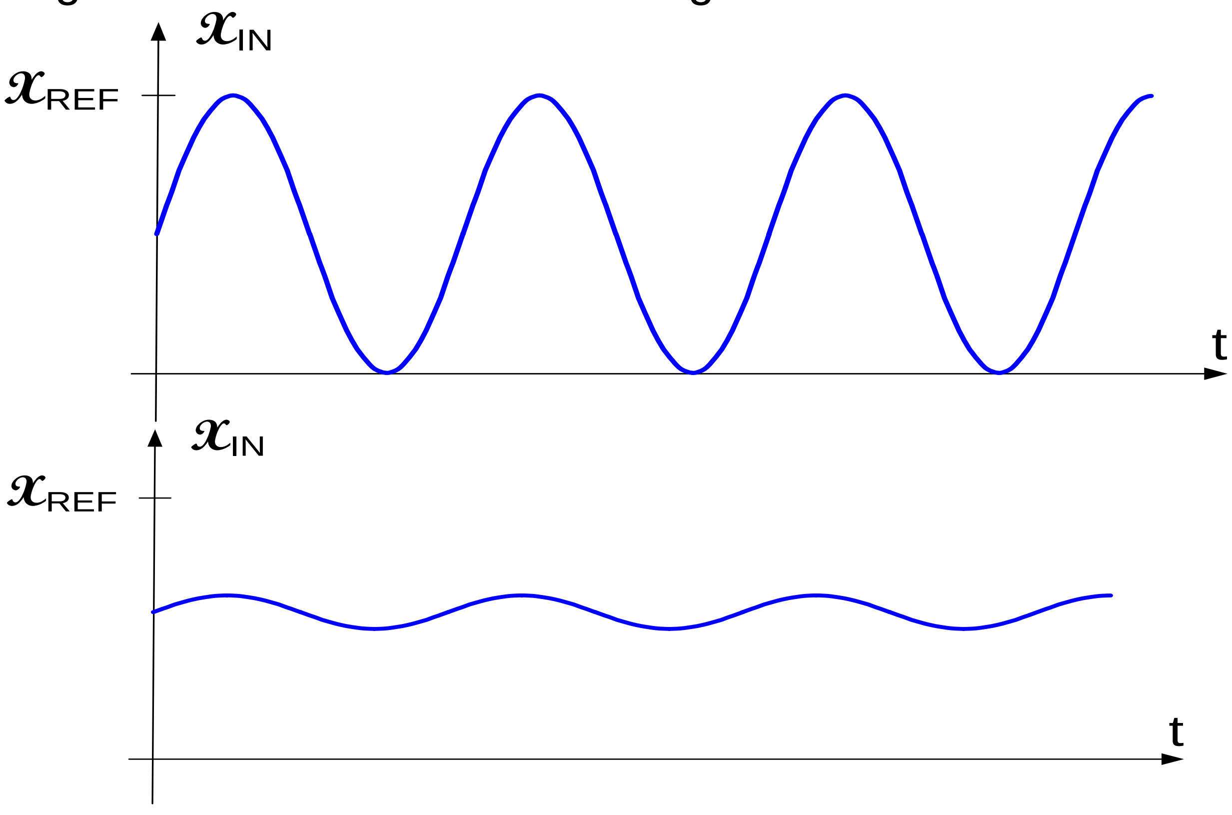
$SNR = 6.02\, n + 1.76$

Table values in dB

Almost no difference for $n \geq 3$

Quantization Noise

Effects of quantization noise can be very significant, even at high resolution, when signals are not of maximum magnitude

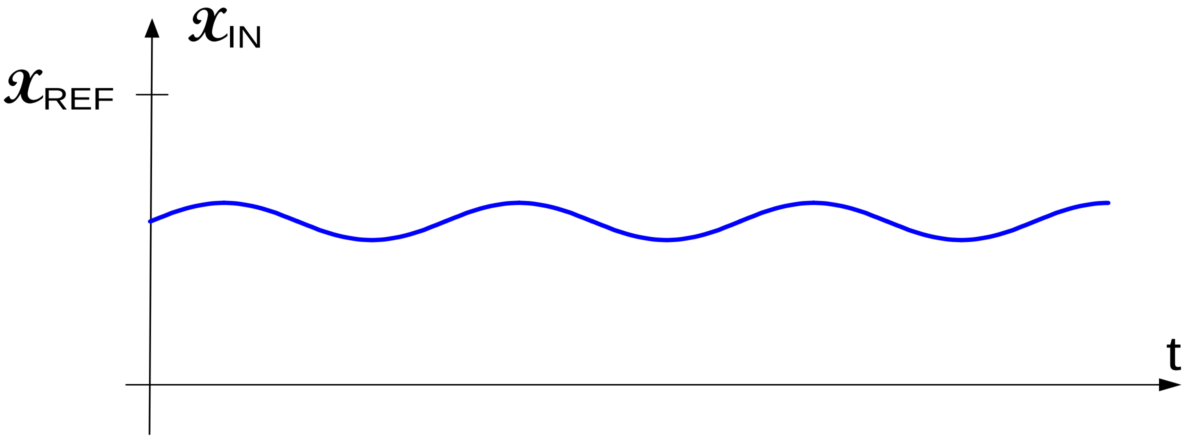


Quantization noise remains constant but signal level is reduced

The desire to use a data converter at a small fraction of full range
is one of the major reasons high resolution is required in many applications

Quantization Noise

Effects of quantization noise can be very significant, even at high resolution, when signals are not of maximum magnitude



Quantization Noise

Example: If a 14-bit audio output is derived from a DAC designed for providing an output of 100W but the normal listening level is at 50mW, what is the SNR due to quantization noise at maximum output and at the normal listening level? What is the ENOB of the audio system when operating at 50mW?

At 100W output, SNR=6.02n+1.76 = 90.6dB

$$\frac{V^2}{R_L}=100\text{W} \qquad \frac{V_1^2}{R_L}=50\text{mW} \qquad V_1=\frac{V}{44.7}$$

$$20\log_{10}V_1=20\log_{10}V-20\log_{10}44.7=20\log_{10}V\ \ -33\text{dB}$$

At 50mW output, SNR reduced by 33dB to 57.6dB

$$\text{ENOB} = \frac{\text{SNR}_{\text{dB}}-1.76}{6.02} = \frac{57.6-1.76}{6.02} = -9.3$$

Note the dramatic reduction in the effective resolution of the DAC when operated at only a small fraction of full-scale.

ENOB Summary

Resolution:

$$\text{ENOB} = \frac{\log_{10} N_{ACT}}{\log_{10} 2} = \log_2 N_{ACT}$$

INL:

$$\text{ENOB} = n_R - \log_2(\upsilon) - 1$$

n_R specified res, v INL in LSB

$$\text{ENOB} = -\log_2(\text{INL}_{REF}) - 1$$

INL_{REF} INL rel to X_{REF}

DNL:

HW problem

Quantization noise:

rel to triangle/sawtooth

$$\text{ENOB} = \frac{\text{SNR}_{dB}}{6.02}$$

$$\text{ENOB} = \frac{\text{SNR}_{dB} - 1.76}{6.02}$$

rel to sinusoid

Most widely used for static characteristics

Additional ENOB will be introduced when discussing dynamic characteristics

Performance Characterization of Data Converters

- Static characteristics
  - Resolution
  - Least Significant Bit (LSB)
  - Offset and Gain Errors
  - Absolute Accuracy
  - Relative Accuracy
  - Integral Nonlinearity (INL)
  - Differential Nonlinearity (DNL)
  - Monotonicity (DAC)
  - Missing Codes (ADC)
  - Quantization Noise
  - Low-f Spurious Free Dynamic Range (SFDR)
  - Low-f Total Harmonic Distortion (THD)
  - Effective Number of Bits (ENOB)
  - Power Dissipation

Absolute Accuracy

Absolute Accuracy is the difference between the actual output and the ideal or desired output of a data converter

The ideal or desired output is in reference to an absolute standard (often maintained by the National Institute of Standards and Technology – NIST) (renamed from National Bureau of Standards in 1988) and could be volts, amps, time, weight, distance, or one of a large number of other physical quantities)

Absolute accuracy provides no tolerance to offset errors, gain errors, nonlinearity errors, quantization errors, frequency rolloff, or noise

In many applications, absolute accuracy is not of a major concern

Absolute accuracy generally dominated by the nonidealities of the reference (a data converter is a ratio-metric device so no fundamental limit on ratio portion)

but … scales, meters, etc. may be more concerned about absolute accuracy than any other parameter

Relative Accuracy

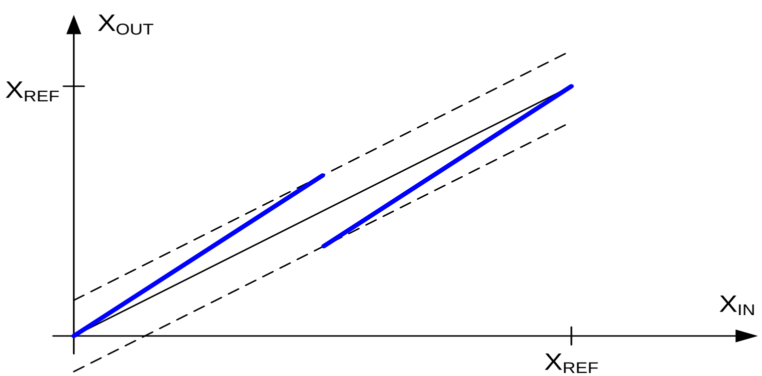
In the context of data converters, pseudo-static Relative Accuracy is the difference between the actual output and an appropriate fit-line to overall output of the data converter

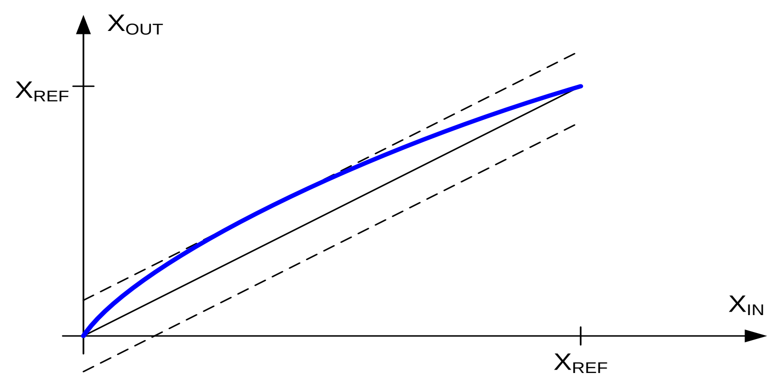
INL is often used as a measure of the relative accuracy

In many, if not most, applications, relative accuracy is of much more concern than absolute accuracy

Some architectures with good relative accuracy will have very small deviations in the outputs for closely-spaced inputs whereas others may have relatively large deviations in outputs for closely-spaced inputs

DNL provides some measure of how outputs for closely-spaced inputs compare





Performance Characterization of Data Converters

- Static characteristics
  - Resolution
  - Least Significant Bit (LSB)
  - Offset and Gain Errors
  - Absolute Accuracy
  - Relative Accuracy
  - Integral Nonlinearity (INL)
  - Differential Nonlinearity (DNL)
  - Monotonicity (DAC)
  - Missing Codes (ADC)
  - Quantization Noise
  - Low-f Spurious Free Dynamic Range (SFDR)
  - Low-f Total Harmonic Distortion (THD)
  - Effective Number of Bits (ENOB)
  - Power Dissipation

Limitations of INL & DNL in Characterizing Linearity

- **INL is a key parameter that is attempting to characterize the overall linearity of a DAC !**

- **INL is a key parameter that is attempting to characterize the overall linearity of an ADC !**

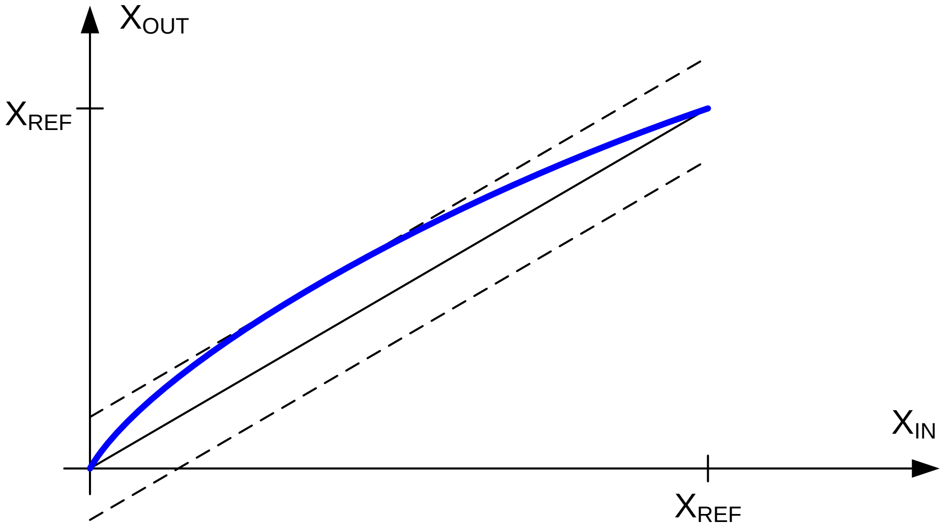
- **DNL is a key parameter that is attempts to characterize the local linearity of a DAC !**

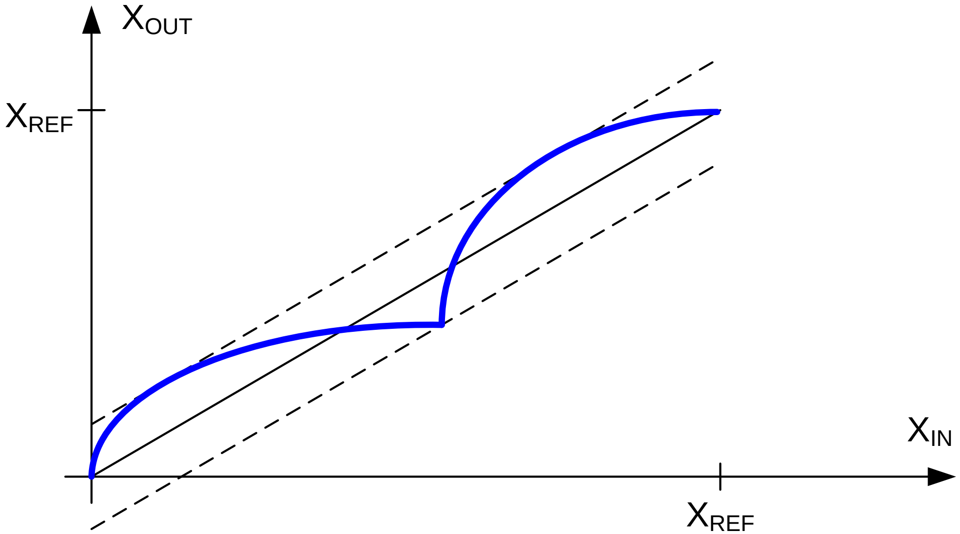
- **DNL is a key parameter that is attempts to characterize the local linearity of an ADC !**

Are INL and DNL effective at characterizing the linearity of a data converter?

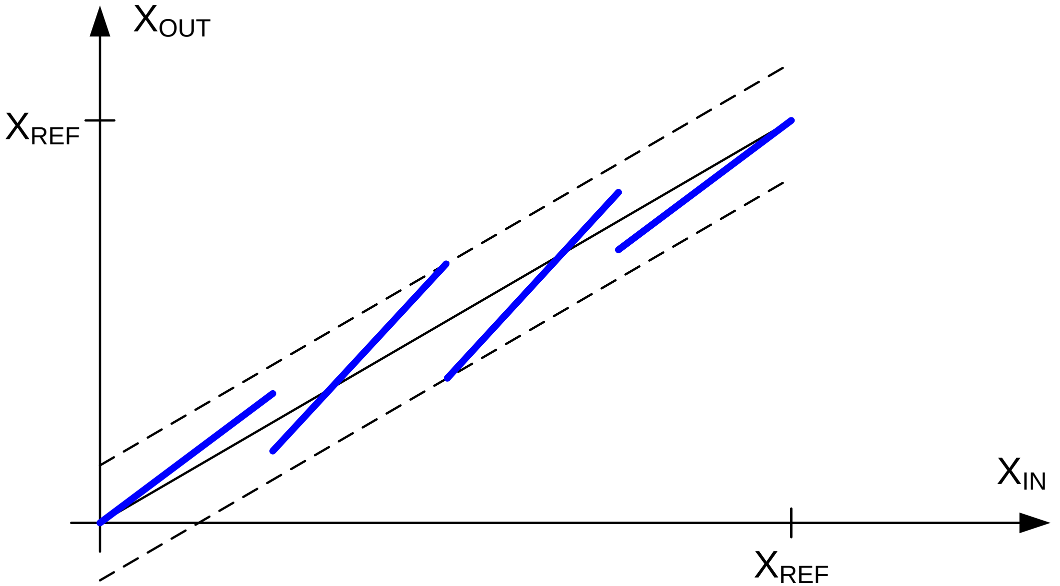
Limitations of INL & DNL in Characterizing Linearity

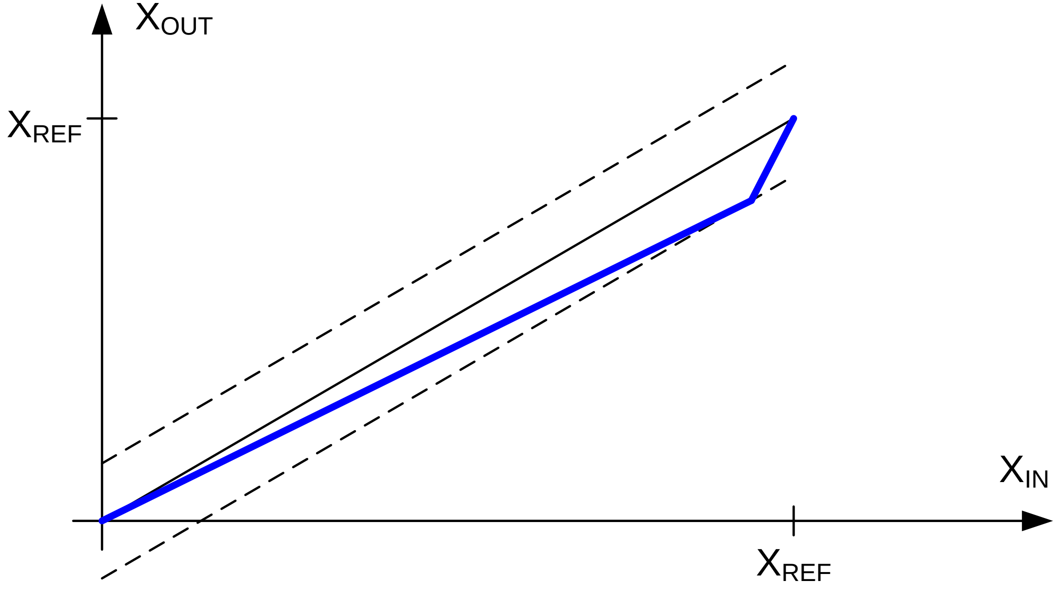
Consider the following 4 transfer characteristics, all of which have the same INL



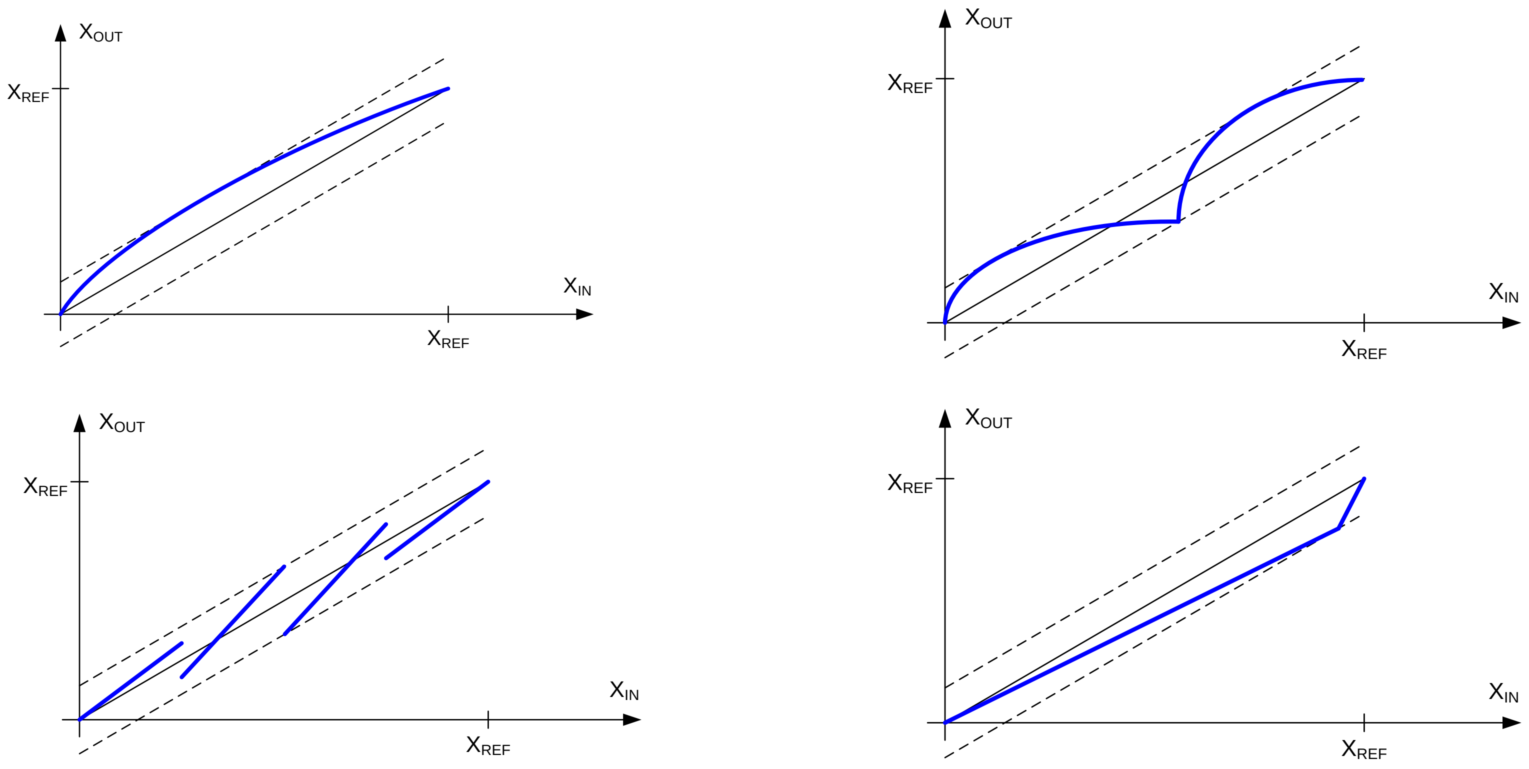


Limitations of INL & DNL in Characterizing Linearity





Limitations of INL & DNL in Characterizing Linearity



Although same INL, dramatic difference in performance particularly when inputs are sinusoidal-type excitations

INL also gives little indication of how performance degrades at higher frequencies

Spectral Analysis often used as an alternative (and often more useful in many applications) linearity measure for data converters

Linearity Issues

- INL is often not adequate for predicting the linearity performance of a data converter
- Distortion (or lack thereof) is of major concern in many applications
- Distortion is generally characterized in terms of the harmonics that may appear in a waveform

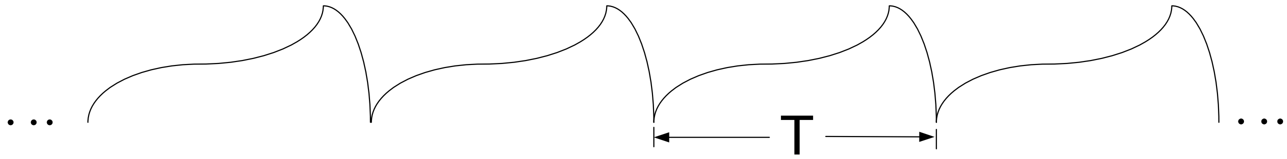
Spectral Analysis often used as an alternative (and often more useful in many applications) linearity measure for data converters

Performance Characterization of Data Converters

- Static characteristics
  - Resolution
  - Least Significant Bit (LSB)
  - Offset and Gain Errors
  - Absolute Accuracy
  - Relative Accuracy
  - Integral Nonlinearity (INL)
  - Differential Nonlinearity (DNL)
  - Monotonicity (DAC)
  - Missing Codes (ADC)
  - Quantization Noise
  - Low-f Spurious Free Dynamic Range (SFDR)
  - Low-f Total Harmonic Distortion (THD)
  - Effective Number of Bits (ENOB)
  - Power Dissipation

Spectral Characterization

Spectral Analysis



If a function g(t) is periodic

$$g(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin\left(k\omega t + \theta_k\right)$$

alternately

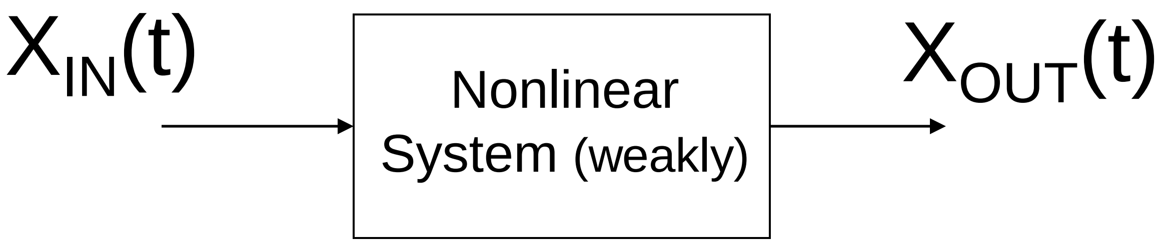
$$g(t) = A_0 + \sum_{k=1}^{\infty} a_k \sin\left(k\omega t\right) + \sum_{k=1}^{\infty} b_k \cos\left(k\omega t\right)$$

$$\omega = \frac{2\pi}{T}$$

$$A_k = \sqrt{a_k^2 + b_k^2}$$

Termed the Fourier Series Representation of g(t)

Spectral Analysis



Often the system of interest is ideally linear but practically it is weakly nonlinear.

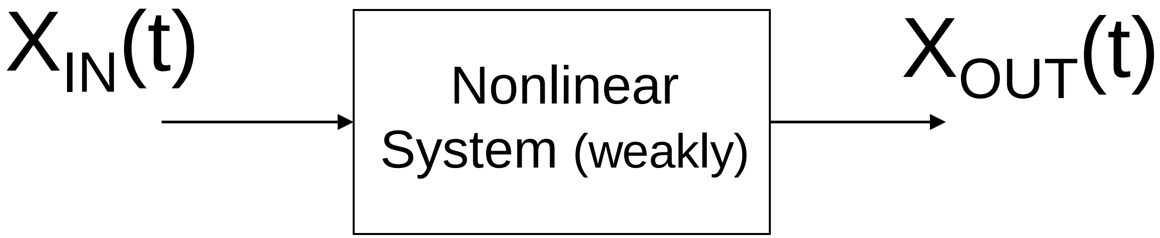
Often the input is nearly periodic and often sinusoidal and in latter case desired output is also sinusoidal

Weak nonlinearity will cause harmonic distortion (often just termed distortion) of signal as it is propagated through the system

Spectral analysis often used to characterize effects of the weak nonlinearity

- **Spectral analysis is another approach the is widely used to characterizing nonlinearity of a data converter (or an arbitrary system)**

Spectral Analysis



Distortion Types:

Frequency Distortion

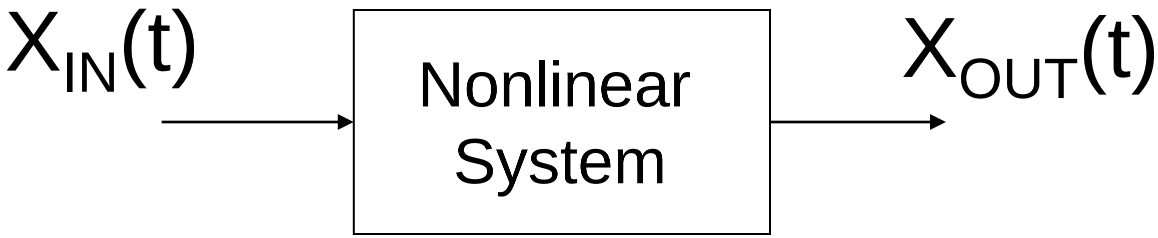
Nonlinear Distortion (alt. harmonic distortion)

Frequency Distortion: Amplitude and phase of system is altered but output is linearly related to input

Nonlinear Distortion: System is not linear, frequency components usually appear in the output that are not present in the input

Spectral Analysis is the characterization of a system with a periodic input with the Fourier series relationships between the input and output waveforms

Spectral Analysis



If $$X_{IN}(t) = X_m \sin(\omega t + \theta)$$

$$X_{OUT}(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin(k\omega t + \theta_k)$$

All spectral performance metrics depend upon the sequences $\langle A_k \rangle_{k=0}^{\infty}$ $\langle \theta_k \rangle_{k=1}^{\infty}$
(index sequence, not time sequence)

Typical spectral performance metrics of interest: SNDR, SDR, THD, SFDR, IMOD

Alternately

$$X_{OUT}(t) = A_0 + \sum_{k=1}^{\infty} a_k \sin(k\omega t) + \sum_{k=1}^{\infty} b_k \cos(k\omega t) \qquad A_k = \sqrt{a_k^2 + b_k^2} \qquad \theta_k = \tan^{-1}\left(\frac{b_k}{a_k}\right)$$

Stay Safe and Stay Healthy !

End of Lecture 4